Constrained load/frequency control problems in networked multi-area power systems

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Abstract

In this paper, we present a supervisory discrete-time predictive control strategy for load/frequency control problems in networked multi-area power systems subject to coordination constraints. Coordination between the control center and the spatially distributed areas is accomplished via data networks subject to communication latency modeled by time-varying time-delay. The aim here is finding supervising strategies able to reconfigure, whenever necessary in response to unexpected load changes and/or faults, the nominal set-points on frequency and generated power to the generators of each area so that viable evolutions would arise for the overall power system and a new sustainable equilibrium is reached. In order to demonstrate the effectiveness of the strategy, examples on a four-area power system are presented.

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1. Introduction

In a restructured system, power systems consist of several interconnected control areas where each one is responsible for its native load and scheduled interchanges with neighboring areas. A hierarchical control structure, with many local and some wide-area controllers is required to effectively handle all adverse phenomena, such as imbalances, fluctuations, disturbances, etc.

Typical local control objectives include: power system protection [1], power production stabilization [2], local voltage control and power flow control. As a common feature,
a regional communication infrastructure is required to be effective because their actions depend on data spread out in the power grid at far away locations.

Load frequency control (LFC) is the mechanism by which a balance between power generation and demand is satisfied. Its basic objective is the global matching between power generation and demand, which must be maintained as close as possible regardless of load fluctuations. If an imbalance results, large frequency deviations may occur with serious impacts on system operations. Power imbalances are caused by unusual load profiles (see [3,4]), fault/failure events on generating units and/or breakdowns of the tie-lines connecting the utilities amongst the areas. All above phenomena are hard to be anticipated and, if not timely detected and corrected, may cause massive power shortages or excesses which result in unacceptable frequency deviations exceeding minimum and maximum limits [5]. This usually leads to protection relays activation with the corresponding disconnection of many components of the power grid and the breakup of the entire system into relatively small disjoined islands.

Typical LFC actions are finalized to maintain the frequency deviations within acceptable and secure limits during these events. However, it is fair commenting that the LFC scope is limited to the stabilization of small imbalances around the nominal equilibrium. In fact, these controllers are not usually capable, for example, of taking the system from a pre-fault equilibrium and steering it into another stable post-fault equilibrium, outside the stability region of the former [6]. Current LFC architectures become ineffective during extreme events (e.g. huge power demand or tie-lines breakdown) and, as a consequence, undesirable transient phenomena could compromise the system integrity if the action of the tripping devices would not take place.

Details on LFC can be found e.g. in [5–14]. The recent survey [15] offers an update and complete overview of LFC strategies. Moreover, a detailed discussion on technical ancillary services and economic features strictly related to frequency and voltage control problems in power systems around the world is given in the two surveys [16,17]. In essence, the LFC requirements are satisfied by the so-called ALFC (automatic load frequency control) stage, which is implemented in the primary and secondary control layers. The primary loop provides a basic small-signal compensation and is computed on the basis of local information only. While, the secondary LFC stage is implemented mainly for that purpose and is in charge to regulate frequency deviations by collecting, via dedicated data links, information on frequencies and power flows at relevant points of the grid and generate, usually in a centralized fashion (in a central station), appropriate counter-actions.

An important aspect in networked applications, rarely considered in the load/frequency control literature, is how to take care of the communication latency affecting the data links connecting each area to the control center. In this paper, the network latency is considered and it is abstractly modeled by a time-varying time-delay. If the network’s protocol is equipped with a time-stamping mechanism, the linear time-invariant discrete-time (LTI-TD) paradigm can be used for system description. See [18] and references therein for further discussions and theoretical justifications on this point.

The necessity to face with the problem of delayed communication is receiving increasing importance from the researcher community and it is now well understood that time-delay can compromise the system integrity by driving the power system toward the instability or other unacceptable behaviors when authoritative wide-area control actions are implemented. Currently, only few recent works (see [6,7,9,14]) have analyzed the presence of the
communication time-delay in power system networks, and limitedly to normal operating
scenarios. In this paper we propose an on-line tertiary LFC supervisory approach based on
constrained control ideas, initially developed for telecontrol problems [19], aimed at
supervising remotely located generation units, connected by distribution lines to remotely
distributed loads, under coordination requirements consisting of enforcing pointwise-in-time
constraints on the evolutions of relevant system variables. In particular, a LFC scheme able
to directly enforce constraints on maximum allowable frequency and power deviations from
nominal values is proposed, and it will be shown that the scheme is also effective in the
presence of time-varying communication time-delays and possible data-loss.

We focus on the master/slaves scenario depicted in Fig. 1, where the ALFC master
represents a supervisor in charge of taking decisions based on delayed information coming
from the areas via data networks, namely suitable state vectors \( x_i(t-\tau) \), with \( \tau \) being
the time-delay representing the network latency. At the remote sides, the ALFC slave units are
those parts of the strategy which require only local, though updated, information. Then,
they are implemented distributively in the proximity of each generating unit (for simplicity
we will assume hereafter that a single generating unit per area exists). Primary and
secondary standard control structures are still present and are not modified by the addition
of this proposed tertiary control level.

In Fig. 1, the signals \( r_i, d_i, x_i, y_i \) and \( c_i \) represent respectively: nominal set-points, loads,
states, performance-related and coordination-related outputs for the \( i \)-th area. In such a
case, the supervisory task can be expressed as the requirement of satisfying some
tracking performance, viz. \( y_i(t) \approx r_i(t) \), whereas the coordination task consists of enforcing
pointwise-in-time constraints of the form \( c_i(t) \in C_i \) on each area and/or \( f(c_1(t), c_2(t), \ldots, c_N(t)) \in C \)
on the overall evolution of the power-grid. These constraints can be used to
impose all safety, operative and coordination requirements that ensure feasible evolutions
to the power-grid so as to prevent the actions of the tripping devices from occurring.

For each area, the ALFC master action consists of a vector \( \hat{z}_i := [\hat{g}_i \hat{y}_i] \), where \( \hat{g}_i \)
represents the best approximation of \( r_i \) compatible with the prescribed constraints. The
modification of \( r_i \) in \( g_i \) takes place if the equilibrium corresponding to \( r_i \) is no longer
sustainable given the actual changed conditions. As a result, if \( g_i \) is applied instead of \( r_i \), it
does not produce constraints violation and allows the overall network to reach a new
feasible and sustainable equilibrium. The vector \( \hat{y}_i \) is an additional optional command.

![Fig. 1. A distributed tertiary LFC structure for a two-area power system.](image)
If used, it represents an offset sequence to be added at the input terminal of the generation unit in order to enlarge the set of feasible evolutions for that area and for the overall power grid. It is worth commenting that, because of time-delay, the computed vectors $\hat{z}_i(t, t+\tau)$ are double indexed. Such a notation indicates that they are computed at time $t$ for being applied exactly at time $t+\tau$ or nevermore. It has been shown [22] that their exact application at time $t+\tau$ is mandatory to ensure constraints fulfilment and one cannot depart from. Finally, because of possible data-loss, if the vector $\hat{z}_i(t-\tau, t)$ is not available at time $t$ at the remote side, the ALFC slave logic is typically instructed to replace it with the last applied vector $z_i(t-1)$. Then, we denote with $z_{i(t)}$ the vector of commands really applied at time $t$ and allow it to be different from $\hat{z}_i(t-\tau, t)$ in general.

The above ALFC scheme makes use of a LTI-TD dynamic model of each area for computing the future state predictions of the power grid. Moreover, unlike most LFC schemes reported in the literature, here the mathematical models of the power grid are defined in terms of absolute variables. The rationale of such a choice hinges upon the possibility of taking advantage of set-point adjustments w.r.t. their nominal values in order to enlarge the set of feasible transients. This is of paramount importance in the presence of faults and/or unexpected large load changes for avoiding, as long as possible, the intervention of tripping devices.

It is expected that the above described ALFC strategy, in response to critical events, is capable to reconfigure the set-points and the additional offsets for the local primary/secondary control laws in such a way to maintain the overall system evolutions always coordinated, within the prescribed safety and operative constraints.

The core of the proposed supervisory strategy is based on a predictive control approach used recently to synthesize reference governor (RG) [20–24] and parameter governor (PG) units [25]. The idea here is to combine in a single unit both strategies referred hereafter to as reference-offset governor (ROG). Moreover, in order to take care of the time-varying communication latency possibly existing from the control center to each area, an extension of this strategy is presented, hereafter referred to as distributed reference offset governor (DROG), by resorting to the master/slaves ideas of [22].

2. Power system model and operative constraints

In this section, a two-area power system and its load/frequency control scheme (see Fig. 2) is detailed.

By referring to a single area, a small-signal dynamical model of the two-area power system is as follows:

$$\dot{f}_i(t) = -\frac{1}{T_{P_i}}(f_i(t)-f_{iref}) + \frac{K_{P_i}}{T_{P_i}} P_{T_i}(t) - \frac{K_{P_i}}{T_{P_i}} P_{tie}(t) - \frac{K_{P_i}}{T_{P_i}} \left( P_{D_i}(t)-\frac{f_{iref}}{K_{P_i}} \right)$$

$$\dot{P}_{T_i}(t) = -\frac{1}{T_{T_i}} P_{T_i}(t) + \frac{1}{T_{T_i}} P_{v_i}(t)$$

$$\dot{P}_{v_i}(t) = -\frac{1}{R_{i}T_{G_i}} (f_i(t)-f_{iref}) - \frac{1}{T_{G_i}} P_{v_i}(t) - \frac{1}{T_{G_i}} P_{c_i}(t) + \frac{1}{T_{G_i}} \theta_i(t)$$

$$\dot{P}_{c_i}(t) = -K_i (P_{tie}(t) + B_{i} (f_i(t)-f_{iref}))$$

$$\dot{P}_{tie}(t) = T_{12}((f_1(t)-f_{1iref})-(f_2(t)-f_{2iref}))$$
where $f_{\text{ref}}$ and $\theta_i$ represent respectively the frequency set-point and the additional offset to be added to the nominal control input. The constant term $f_{\text{ref}}/K_{P_i}$ in Eq. (1) accounts for the representation of the LTI model of the power system in terms of absolute variables (unlike the standard incremental models used more often in the literature [13,26]). Moreover, $P_{T_i}(t)$ is the active power produced by the generator unit, $P_{D_i}(t)$ the local load demand, $P_{v_i}(t)$ the change in the valve position and $P_{c_i}(t)$ the control action, related to the $ACE_i(t)$ error index described below, provided by the secondary control layer for compensating frequency deviations. All the variables are standard and their description can be found in [13].

The task of the primary/secondary control layers is to maintain the power system within acceptable operating limits by achieving the load balance $P_{T_i}(t) \approx P_{D_i}(t)$ while maintaining the area frequency $f_i(t)$ at their nominal value $f_{\text{ref}}$. When the power balance is no longer preserved, because e.g. of a change in the local load $P_{D_i}$ or in the tie-line power of $P_{\text{tie}}(t)$, frequency deviations arise (the frequency $f_i(t)$ shifts w.r.t. its set-point). The effect for customers is a lower quality of the delivered electrical energy and damaging vibrations in the turbines (at frequencies lower than $57 - 58$ Hz) may occur.

The load/frequency control requirements are traditionally satisfied by the ALFC stage (primary and secondary loops), which typically takes the form of a PI local controller. The purpose of the primary loop is to achieve fast adjustments on the produced power $P_{T_i}(t)$ in response to local frequency deviations. To this end, by means of the regulation parameter $R_i$, the following control action:

$$P_{c_i}(t) = \frac{1}{R_i} (f_i(t) - f_{\text{ref}})$$

(6)
is fed into the governor unit, here represented by a first order lag $1/(1 + sT_G)$. Such a command causes a position change $P_v(t)$ of the control valve which, in turn, translates into a power increment of $P_T(t)$.

The secondary ALFC action is based on an error signal, referred to as area control error (ACE), which in addition to local frequency deviations also contains information on global frequency deviations via the power exchanged between the areas via the tie-line [26]. For each $i$-th area, the ACE index is defined as

$$ACE_i(t) = P_{tie}(t) + B_s(f_i(t) - f_{ref})$$

(7)

and the command $P_v(t)$ is computed as shown in Eq. (4). Finally, the block $K_p/(1 + sT_p)$ is used to model the linear power system dynamics (inertia constants and damping coefficients).

The above ALFC loops perform in a satisfactory way under small power imbalances arising during normal operating conditions. On the contrary, when the power system is in an emergency state due to a sudden generator loss or to a tie-line disconnection, the ALFC stage is ineffective because turbines have slow responses. In practice, for handling such situations, the power system is equipped with the so-called emergency control whose task is to recover the normal operating state. However, such efforts could fail and the system would end up in a total blackout.

2.1. Operative constraints

Our approach in trying to mitigate the occurrence of the above abnormal scenarios is that of defining a set of operative and safety constraints, which characterize more precisely the normal conditions one would like to impose, to be fulfilled as longer as possible by the system evolutions under the action of the proposed tertiary supervisory scheme. To this end, the following constraints will be considered:

$$f_{min,i} \leq f_i(t) \leq f_{max,i}, \quad i = 1, 2$$

(8)

$$|P_{tie}(t)| \leq \beta_{max}$$

(9)

$$\gamma_{min,i} \leq P_{T_i}(t) \leq \gamma_{max,i}, \quad i = 1, 2$$

(10)

Specifically, constraint (8) prescribes bounds on maximum frequency deviations for the $i$-th area, Eq. (9) imposes bounds on the power flow exchanged between the areas via the tie-line, while Eq. (10) limits the maximum generable power in each area. The latter springs from the existing limitations on each generation unit, which can be identified as a saturation on the power production.

3. Distributed reference-offset governor (DROG) device

In this section, the main ideas and properties of the proposed master/slaves DROG approach will be presented. This solution is a direct generalization of the GC scheme presented in [29]. For the sake of clarity, the approach is presented for a single slave system (a single area). The extension to the multi-slaves case is straightforward and follows similar ideas of [22]. Consider the master/slave scenario depicted in Fig. 3.
The typical system structure for the slave side consists of a primal compensated plant (dashed box) described by the following state-space representation:

\[
\begin{align*}
    x(t+1) &= \Phi x(t) + Gz(t) + G_d d(t) \\
    y(t) &= H_y x(t) \\
    c(t) &= H_c x(t) + Lz(t) + L_d d(t)
\end{align*}
\]  

(11)

with \( t \in \mathbb{Z}_+ = \{0, 1, \ldots\}; \) \( x(t) \in \mathbb{R}^n \) the state vector (which includes the controller states if any); \( z(t) = [z(t)] \in \mathbb{R}^{2m} \) the ROG output, where \( g(t) \in \mathbb{R}^m \) is the manipulable reference which, if no constraints were present, it would essentially coincide with the reference \( r(t) \in \mathbb{R}^n \). Moreover, \( \theta(t) \in \mathbb{R}^m \) is an adjustable offset on the nominal control input which we assume to be selected from a given convex and compact set \( \Theta \), with \( 0_m \in \text{int} \Theta; \) \( d(t) \in \mathbb{R}^m \) an exogenous bounded disturbance satisfying \( d(t) \in \mathcal{D}, \forall t \in \mathbb{Z}_+ \) with \( \mathcal{D} \) a specified convex and compact set such that \( 0_m \in \mathcal{D}; \) \( y(t) \in \mathbb{R}^m \) the output, viz. a performance related signal; and finally \( c(t) \in \mathbb{R}^n \) the constrained vector

\[
c(t) \in \mathcal{C}, \quad \forall t \in \mathbb{Z}_+
\]

(12)

with \( \mathcal{C} \subset \mathbb{R}^n \) a prescribed convex and compact set. Moreover, the following matrices are defined

\[
G = \begin{bmatrix} G_y & G_\theta \end{bmatrix}, \quad L = \begin{bmatrix} L_y & L_\theta \end{bmatrix}
\]

It is assumed that

\[
\begin{align*}
    (1) & \quad \Phi \text{ is an asymptotically stable matrix}; \\
    (A1) & \quad \text{System (11) is offset – free w.r.t. } g(t) \text{ i.e.} \\
    & \quad H_y (I_n - \Phi)^{-1} G_\theta = I_m
\end{align*}
\]

At the master side, the system representation (11) is used as a model for computing the ROG master action \( \hat{z}(t, t + \tau) = [\hat{g}^T(t, t + \tau) \hat{\theta}^T(t, t + \tau)]^T \in \mathbb{R}^{2m} \) (see Fig. 3), which has to be read as: commands generated at time \( t \) for being applied at time \( t + \tau \). Typically it would be \( z(t) = \hat{z}(t-\tau, t) \) under normal conditions. However, if \( \hat{z}(t-\tau, t) \) would not be available at the remote side at time \( t \) for congestions or data-loss, the ROG slave logic is instructed to apply the previous applied command, viz. \( z(t) = z(t-1) \).

Moreover, it is required that: (1) \( \hat{g}(t, t + \tau) \rightarrow \hat{r} \) whenever \( r(t + \tau) \rightarrow r \), with \( \hat{r} \) the best admissible approximation of \( r \) and \( \hat{\theta}(t, t + \tau) \rightarrow 0_m \); and (2) the DROG have a finite settling time, viz. \( \hat{g}(t, t + \tau) = \hat{r} \) and \( \hat{\theta}(t, t + \tau) = 0_m \) for a possibly large but finite \( t \) whenever the reference stays constant after a finite time.
In order to make our discussion more general, we allow hereafter the time-delay to be possibly time-varying. To this end, let \( \tau_f(t) \) and \( \tau_b(t) \) be the forward and, respectively, backward time-delays at each time instant \( t \) (expressed in sampling units), viz. \( \tau_f(t) \) is the delay from the master to the slave unit whereas \( \tau_b(t) \) is the delay in the opposite direction. We assume further that the following upper-bounds \( \bar{\tau}_f \) and \( \bar{\tau}_b \)

\[
\tau_f(t) \leq \bar{\tau}_f \quad \text{and} \quad \tau_b(t) \leq \bar{\tau}_b
\]

(13) apply.

At each sampling time \( t \), we indicate with \( t_b \) the most recent sampling instant at which the area has sent a piece of information which is received by the ROG master at time instant \( t \) and with \( t_f \) be the most recent sampling instant at which the ROG master has sent a command that is received by the area at time instant \( t \). Then, under Eq. (13) it results that \( t - t_b \leq \bar{\tau}_b \) and \( t - t_f \leq \bar{\tau}_f \), which means that all the delivered signals are timely received. The situation is clarified in Fig. 4.

The idea here is that the DROG master logic device acts as if the time-delay would not be present by modifying, whenever necessary, the reference \( r(t + \tau_f) \) into \( \hat{r}(t, t + \tau_f) \) and by adding an offset \( \hat{\theta}(t, t + \tau_f) \) on the nominal control action. In so doing, at each time \( t \), an admissible command \( \hat{z}(t, t + \tau_f) \) is generated. The computation of the master DROG action relies on the future state and constrained vector predictions from \( t + \bar{\tau}_f \) onward, computed under the assumption that a constant vector \( z \) is applied. To this end, let \( t_b \leq t \) be the most recent sampling instant at which the master has received a piece of information from the slave and \( x(t_b) \) is its state at that time. Notice that the following inequality \( t + \tau_f - t_b \leq \bar{\tau}_b + \bar{\tau}_f \) holds true, which represents the maximum round-trip delay on the network.

By linearity, one is allowed to separate the effects of the initial conditions and inputs from those of disturbances, e.g. \( x(t) = \bar{x}(t) + \tilde{x}(t) \), where \( \bar{x}(t) \) is the disturbance-free component and \( \tilde{x}(t) \) depends only on the disturbances. Then, the remote disturbance-free state at \( t + \bar{\tau}_f \) can easily be computed as

\[
\bar{x}(t + \bar{\tau}_f | t_b) = \Phi^{t + \bar{\tau}_f - t_b} x(t_b) + \sum_{i=t_b}^{t + \bar{\tau}_f - 1} \Phi^{t + \bar{\tau}_f - i - 1} G \hat{z}(i - \tau_f, i)
\]

(14) under the assumption that all commands \( \hat{z} \) delivered from \( t_b \) to \( t + \bar{\tau}_f - 1 \) have been received and timely applied.

In the sequel, we adopt the following notations:

\[
\bar{x}_z := (I_n - \Phi)^{-1} G \hat{z}
\]

\[
\bar{y}_z := H_y (I_n - \Phi)^{-1} G \hat{z}
\]
\[ z(\tau) = \begin{multline} H_c (I_n - \Phi)^{-1} Gz + Lz \end{multline} \] (15)

for the disturbance-free equilibrium solutions of Eq. (11) to a constant command \( z(t) \equiv z \), with \( z(t) \equiv \begin{bmatrix} 0 \end{bmatrix} \), and \( g, \theta \in \mathbb{R}^m \) constant vectors. Consider next the following set of recursion (see Fig. 5 (Left)):

\[
\begin{align*}
C_0 &:= C \sim L_d D \\
C_k &:= C_{k-1} \sim H_c \Phi^{k-1} G_d D \\
&\vdots \\
C_{\infty} &:= \bigcap_{k=0}^{\infty} C_k
\end{align*}
\] (16)

where, for given sets \( A, \mathcal{E} \subset \mathbb{R}^n \), \( A \sim \mathcal{E} \subset A \) is a proper restriction of \( A \) denoted in the literature as the \( P \)-difference between sets [27]:

\[ A \sim \mathcal{E} \equiv \{ x \in \mathbb{R}^n : x + e \in A, \forall e \in \mathcal{E} \} \] (17)

In [23,28] it has been proved that the sets \( C_k \) are non-conservative restrictions of \( C \) such that \( \bar{c}(t) \in C_{\infty}, \forall t \in \mathbb{Z}_+ \), implies that \( c(t) \in C, \forall t \in \mathbb{Z}_+ \). In the above references, it has also been shown that the sets \( C_k \) are the largest restrictions of \( C \) such that \( \bar{c}(t) \in C_k \) implies \( c(t + i) \in C \), any \( d(t + i) \in D, \forall i \in \{0, 1, \ldots, k-1\} \). In other words, if the disturbance-free constrained vector \( \bar{c}(t) \) is contained in \( C_k \) at a certain time instant \( t \), then one is ensured that no constraint violations can occur for the next \( k \) time instants due to disturbances. In particular, \( \bar{c}(t) \in C_{\infty}, \forall t \in \mathbb{Z}_+ \), implies that \( c(t) \in C, \forall t \in \mathbb{Z}_+ \). If \( C_{\infty} \) is empty the problem has no solution. If non-empty, all \( C_k \)'s, \( \forall k \in \mathbb{Z}_+ \), are non-empty, convex, compact and satisfy the nesting condition \( C_k \subset C_{k-1}, \forall k \in \mathbb{Z}_+ \). Thus, one is allowed to consider the disturbance-free system evolutions only and adopt a “worst case” approach. For reasons which have been clarified in [20], it is convenient to introduce the following sets for a given \( \delta > 0 \):

\[ C_{\delta} := C_{\infty} \sim B_{\delta} \] (18)

\[ \mathcal{W}_{\delta} := \{ z \in \mathbb{R}^{2m} : \bar{z}_z \in C_{\delta} \} \] (19)

where \( B_{\delta} \) is a ball of radius \( \delta \) centered at the origin (see Fig. 5 (Right)). Let us assume that there exists a sufficiently small \( \delta > 0 \) such that \( \mathcal{W}_{\delta} \) is non-empty. In particular, \( \mathcal{W}_{\delta} \) is the set of all commands whose corresponding steady-state solution \( \bar{z}_z \) satisfies the constraints with
margin $\delta$. From the foregoing definitions and assumptions, it follows that $\mathcal{W}_\delta$ is closed and convex. Again, emptiness of $\mathcal{W}_\delta$ means that the problem has no solution.

Then, the approach in selecting at each time $t$ the DROG action $z(t)$ will consist of restricting the choice amongst all vectors of a suitable $x(t)$-depending subset of $\mathcal{W}_\delta$, each vector of which, if constantly applied as a command to the system from the time instant $t$ onwards, gives rise to system evolutions which do not produce constraint violations. Notice that the choice $z(t) \in \mathcal{W}_\delta$ only ensures constraints fulfillment in steady-state conditions, viz. when $\overline{e}(t) \in \mathcal{C}_0^d$, but nothing can be said about the transient from $x(t)$ to $x(t)$. If many choices exist, a selection index is used to determine the best feasible approximation to $r$. Such a ROG command is applied, a new state is measured and the procedure is repeated at next time instant $t+1$ on the basis of the new state $x(t+1)$.

In this respect, we consider the following family of constant virtual command sequences:

$$z(\cdot) = \{z(k) \equiv z \in \mathcal{W}_\delta, \forall k \in \mathbb{Z}_+\} \quad (20)$$

and, as a consequence, the disturbance-free state and $c$-vector future predictions emanating from $\overline{x}(t+\tau_f|t_b)$ under a whatever constant command $z$ are given by

$$\overline{x}(k, \overline{x}(t+\tau_f|t_b), z) \equiv \Phi^k \overline{x}(t+\tau_f|t_b) + \sum_{j=0}^{k-1} \Phi^{k-j-1} G z$$

$$\overline{c}(k, \overline{x}(t+\tau_f|t_b), z) \equiv H \overline{x}(k, \overline{x}(t+\tau_f|t_b), z) + L z$$

$$(21)$$

where $\overline{c}(k, \overline{x}(t+\tau_f|t_b), z)$ has to be understood as the disturbance-free virtual evolution at virtual time instant $k$ (opposite to the real time $t$) of the constrained vector $c$, from the initial condition $x(t)$ (applied at virtual time zero) under the constant command $z(\cdot) \equiv z$. Finally, we define the set $\mathcal{V}(\overline{x}(t+\tau_f|t_b))$ as

$$\mathcal{V}(\overline{x}(t+\tau_f|t_b)) = \{z \in \mathcal{W}_\delta : \overline{c}(k, \overline{x}(t+\tau_f|t_b), z) \subset C_{k+\tau_b+\tau_f}, \forall k \in \mathbb{Z}_+\} \quad (22)$$

collects all constant virtual commands in $\mathcal{W}_\delta$ whose corresponding $c$-evolutions starting at time $t+\tau_f$ from the state $\overline{x}(t+\tau_f|t_b)$ satisfy the constraints for all $k \in \mathbb{Z}_+$. This is required for feasibility reasons because we have used the disturbances-free state prediction $\overline{x}(t+\tau_f|t_b)$ instead of $x(t+\tau_f)$, that does not contain information on the disturbances, as initial state in Eq. (21) at each time $t$. However, because of linearity, we can equivalently take care of the overall effect of disturbances on the state evolution from $t_b$ to $t+\tau_f$ in Eq. (14) by restricting the feasibility conditions in Eq. (22) by an amount corresponding to effect of the disturbances acting for a time equal to $\tau_b + \tau_f$, which is an upper-bound of $t + \tau_f - t_b$. Notice that $\mathcal{V}(\overline{x}(t+\tau_f|t_b))$ is finitely determined as proved in [22]. Moreover, because of time-invariance, the above machinery (20)–(22) can be used at each time instant $t$. As a consequence $\mathcal{V}(\overline{x}(t+\tau_f|t_b)) \subset \mathcal{W}_\delta$, which, if non-empty, represents the set of all constant virtual sequences in $\mathcal{W}_\delta$ whose evolutions starting from $x(t)$ satisfy the constraints also during transients. In [24], it has been proved that such a set is finitely determined, viz. there exists a positive integer $k_0$ such that Eq. (22) is identically characterizable by restricting $k \in \{0, \ldots, k_0\}$, with $k_0$ computable off-line.

Then, provided that $\mathcal{V}(\overline{x}(t+\tau_f|t_b))$ is non-empty, closed and convex at each time $t \in \mathbb{Z}_+$, the DROG master output is chosen according to the solution of the following constrained optimization problem:

$$\hat{z}(t, t+\tau_f) = \arg \min_{z \in \mathcal{V}(\overline{x}(t+\tau_f|t_b))} \|g - r(t+\tau_f)\|_\psi^2 + \|\theta\|_\psi^2$$

$$(23)$$
where $\Psi_{\theta} = \Psi_{\theta}^T > 0_m$, $\Psi_\gamma = \Psi_\gamma^T > 0_m$ and $\|\psi\| = \psi^T \psi$. It represents the best feasible approximation of $r(t)$, which, if constantly applied from $t$ onwards to the system, would never produce constraints violation. The slave part of the DROG logic is far more simpler and reduces to

$$z(t) = \hat{z}(t-\tau_f, t)$$

(24)

This strategy is justified by the fact that $\mathcal{V}(\overline{\mathcal{X}}(\tau_f|0))$ non-empty implies that $\mathcal{V}(\overline{\mathcal{X}}(t+\tau_f|t_0))$ is non-empty along system evolutions generated by the DROG strategy (23)-(24). The main properties of the DROG strategy may be summarized in Theorem 1.

**Theorem 1.** Consider system (11) along with the DROG logic (23)-(24). Let assumptions (A1) be fulfilled and $\mathcal{V}(\overline{\mathcal{X}}(t_0 + \tau_f|t_0))$ be non-empty, where $t_0$, is the time instant at which the master receives the first piece of information from the slave, generated at time $t_0 \leq t_0$. Then

1. The minimizer in Eq. (23) uniquely exists at each $t \in \mathbb{Z}_+$ and can be obtained by solving a convex constrained optimization problem, viz. $\mathcal{V}(x(\tau_f|0))$ non-empty implies $\mathcal{V}(x(t + \tau_f|t_0))$ non-empty along the trajectories generated by the DROG logic (23)-(24).
2. The set $\mathcal{V}(\overline{\mathcal{X}}(t + \tau_f|t_0))$, $\overline{\mathcal{X}}(t) \in \mathbb{R}^n$, is finitely determined, viz. there exists an integer $k_0$ such that if $\tau(k, \overline{\mathcal{X}}(t + \tau_f|t_0), z) \in C_k$, $k \in \{0, 1, \ldots k_0\}$, then $\tau(k, \overline{\mathcal{X}}(t + \tau_f|t_0), z) \in C_k \forall k \in \mathbb{Z}_+$. Such a constraint horizon $k_0$ can be determined off-line.
3. The constraints are fulfilled for all $t \in \mathbb{Z}_+$.
4. The overall system is asymptotically stable; in particular, whenever $r(t) \equiv r$, $\lim_{t \to \infty} \theta(t) = 0_m$ and $g(t)$ converges either to $r$ or to its best steady-state admissible approximation $\hat{r}$, with

$$\hat{z} := \begin{bmatrix} \hat{r} \\ 0_m \end{bmatrix} := \arg \min_{z \in \mathcal{V}_\delta} J(z, r)$$

where

$$J(z, r) := \|g-r\|_{\Psi_{\theta}}^2 + \|\theta\|_{\Psi_\gamma}^2$$

Consequently, by the offset-free condition (A1)(2), $\lim_{t \to +\infty} \underbar{y}(t) = \hat{r}$, where $\underbar{y}$ is the disturbance-free component of $y$.
5. The strategy achieves a complete time-delay compensation, in that all commands generated at time $t$ at the master side will be applied to the slave with exactly $\tau_f$ sampling steps of delay.

**Proof.** The proof is straightforward by noting that the DROG logic (23)-(24) essentially implements the ROG scheme of [29] replaced by

$$z(t + \tau_f) = \hat{z}(t, t + \tau_f) = \underbar{z}(\overline{\mathcal{X}}(t + \tau_f), r(t + \tau_f)).$$

Therefore, the properties 1-4 trivially follow. Moreover, because $z(t + \tau_f) = \hat{z}(t, t + \tau_f), \forall t$, the stated time-delay compensation property also follows.

**Remark 1.** It is worth commenting that the previous DROG strategy has been presented for a single area only for the sake of clarity. However, the scheme can directly be applied to any multi-area scenario depicted in Fig. 6. In fact, a four-area system is considered in the final example. In order to give a brief sketch of how generalizing the scheme to an $N$-area power
system let the overall vector \( r = [r_1^T, \ldots, r_N^T]^T \) collect all references \( r_i \) to each \( i \)-th area and so on for the other overall vectors of interest, i.e. the command vector \( w = [w_1^T, \ldots, w_N^T]^T \), the states vector \( x = [x_1^T, \ldots, x_N^T]^T \), the performance output vector \( y = [y_1^T, \ldots, y_N^T]^T \) and the constraints vector \( c = [c_1^T, \ldots, c_N^T]^T \). Then, at the ROG master side, all \( N \)-area models and their primal controllers are used for computing the future state predictions of the overall power system. In particular, all future state predictions are updated as soon as a new piece of information is received from a whatever single area. Therefore, the computation of the sets \( \mathcal{W}_b \) and \( \mathcal{V}(x(t + \tau_f | t_b)) \) follows exactly the same lines as before with the obvious noticeable difference that the computational complexity increases with \( N \). Details can be found in [22].

Remark 2. Finally, it is worth to point out that the ideas developed here could be arranged for implementation in a distributed way. Along this direction, a preliminary contribution is given in [33], where a distributed supervision strategy for multi-agent linear systems connected via data networks and subject to coordination constraints is proposed. Applications to distributed LFC problems have been reported in [34].

4. Simulations

The aim of this section is to analyze the behavior of the supervised networked power system and to verify the capabilities of the proposed tertiary LFC scheme to reconfigure the frequency set-points and control offsets despite large load deviations from nominal conditions and/or the occurrence of faults.

In the numerical simulations, the linear turbine model has been enriched by considering a saturation effect (±0.1 limits) on the rate of change of the produced power (see Fig. 9). Such a static nonlinear device has been introduced to take into account practical limitations in the turbine response. Moreover, the model has been also equipped with saturation devices on the produced powers in order to explicitly take into account their physical limits (10).
In all forthcoming simulations we have used identical forward and backward random time-varying delay bounded by $\overline{\tau}_f = \overline{\tau}_b = 2$ (sampling steps), depicted in Fig. 7. Though the delay changes significantly in the figure before and after time instant $t = 110$ s, it will result that the performance is quite independent on the actual value of the delay and depends only on the assumed upper-bounds, the lower the better.

For a detailed discussion on the constraints (8)–(10), the interested reader is referred to [29].

The full nonlinear model depicted in Fig. 9 is considered in the simulations for assessments. Notice however that the system evolution under the DROG action essentially coincides with that obtained by using a LTI model of the power system. This hinges upon the fact that the power system remains within its linear regime when all constraints are fulfilled and the time-delay is bounded.

Coordination of a four-area power system will be considered in the simulations. Comparisons with traditional strategies not based on the use of DROG units will be also considered. All simulations have been carried out with Matlab 7.0 and Simulink 6.0.

In the sequel, an interconnected four-area power system has been used to confirm the effectiveness of the proposed approach in a more realistic power system scenario. A simplified representation for an interconnected system in a general form, including both ring [8] and bus [10] interconnections, is shown in Fig. 8. In particular, only Area 1 is depicted in details in Fig. 9, with its interconnections toward the other areas.

The state space model for this system has been described in [13]. For the sake of comprehension, we report the expression for the power exchanged via the tie-lines with the $i$-th area $P_{tie\text{-}i}$

$$P_{tie\text{-}i} = \sum_{j \neq i} \frac{T_{ij}}{s}((f_i - f_{i\text{ ref}}) - (f_j - f_{j\text{ ref}}))\right) = \sum_{j \neq i} P_{tie\text{-}ij}$$ (27)

Then, $P_{tie\text{-}i}$ represents the total power exchanged by the $i$-th area via the tie-lines whereas $P_{tie\text{-}ij}$ the specific contribution between the $i$-th and $j$-th areas. It is important to note that
the above expression is a linear combination, under the weights \( T_{ij} \), of the frequency differences amongst the areas. Clearly, the following power balance is always satisfied:

\[
\sum_i P_{\text{tie } i} = 0
\]

In the simulations, we have assumed that the nominal operating frequency for each area is \( f_{\text{ref}}(t) = 60 \) Hz, \( i = 1, \ldots, 4 \) \( \forall t \). Again, the power demand has the following form \( P_D(t) = \overline{P}_D + \tilde{P}_D(t), i = 1, \ldots, 4 \) where the same nominal load \( \overline{P}_D = 3 \) MW, has been set for each area, while \( \tilde{P}_D(t) \) identifies possible different load variations. Moreover, under nominal conditions, each area has the capability to autonomously balance its own nominal load. In the simulations, we have supposed that power demands could vary up to 33\% with respect to \( \overline{P}_D, \ i = 1, \ldots, 4 \). As a consequence, the following convex and compact region

\[
\mathcal{D}_{P_D} := \{ \tilde{P}_D \in \mathbb{R}^4 : U\tilde{P}_D \leq h \}
\]

where \( U = [I]_4 \) and \( h = [1 1 1 1 1 1 1 1]^T \), characterizes the set of admissible load disturbances (in MW). Observe that such a bound represents the largest disturbance that the DROG may handle. The following set of constraints (expressed in Hz and MW)

\[
\begin{align*}
58.5 & \leq f_i(t) \leq 61.5 \\
|P_{\text{tie } i}(t)| & \leq 0.7 \\
2.2 & \leq P_{T_i}(t) \leq 3.8, \quad i = 1, \ldots, 4
\end{align*}
\]

(29)

to be fulfilled pointwise-in-time are considered. Finally, the following initial condition:

\[
\chi(0) = [60 3 3 3 0 60 3 3 3 0 60 3 3 3 0 60 3 3 3 0 60 3 3 3 0]^T
\]

has been chosen and the initial admissible DROG command vector has been determined as \( g_i(0) = 60, \ i = 1, \ldots, 4 \) and \( \theta_i(0) = 0, \ i = 1, \ldots, 4 \).

We assume for Area 1 the load variations depicted in Fig. 10. It consists first of a step increment of 0.6 MW on the power demand occurring at \( t = 2 \) s. Then, a further increment of 0.25 MW is simulated at \( t = 12 \) s (which is higher than the maximum generable power for the Area 1) and finally, after \( t = 32 \) s, the load demand settles down to 3.4 MW. Here, we focus on a transmission line failure, which seems to be the most interesting example in order to show the benefits coming out from the interconnection among many areas. We
Fig. 9. Block diagram of Area 1.
impose breakdowns on the transmission lines between Area 1 and Areas 2 and 4 during all the simulation (see Fig. 11). It is evident that Area 4 is disconnected from the others. As a consequence, it is not influenced from the remaining network and its exchanged power via the tie-line is always zero (see Fig. 13).

In order to clarify the power system response, we restrict our attention to the highest load request (3.85 MW), which overcomes the maximum power production of Area 1 (see constraints (29)). The corresponding system evolutions are reported in Figs. 12–14.

In the faulty network topology of Fig. 11, Area 1 is directly connected only to Area 3. Area 1 is not capable to produce the necessary power to balance its local demand and the power fraction required to accomplish the request is furnished via the tie-line $P_{tie1}$. In particular $P_{T1} = 3.575$ MW and $P_{tie1} = 0.275$ MW, while $P_{tie2} = 0.13$ MW, and $P_{tie3} = 0.145$ MW. Recalling that $P_{tie3} = P_{tie31} + P_{tie32}$ and observing that $P_{tie2} = P_{tie23} = -P_{tie32}$, and $P_{tie1} = P_{tie13} = -P_{tie31}$, one can conclude that the overall power balance is
Fig. 12. Generated powers: with DROG.

Fig. 13. Tie-line power flows with DROG.
Fig. 14. Frequencies: with DROG.

Fig. 15. Frequency commands.
satisfied. In fact, as evident from Fig. 12, both Areas 2 and 3 produce more power than their local needs and the surplus contributes to match the demand of Area 1 via the path Area 2 → Area 3 → Area 1. This coincides with a new power routing which can be influenced by the choice of the weighting matrices $\Psi_g$ and $\Psi_\vartheta$ in the cost function (23).

For the sake of completeness, also the frequency evolutions are shown in Fig. 14, where it clearly results that the constraints are never violated. However, some coupling amongst areas is still present as it result from unexpected frequency adjustments in the 4-th Area (see Fig. 14), not interested during the faulty condition to any power exchange. In Figs. 15 and 16 the DROG commands (frequency and offset respectively) are also depicted. Finally, the relationships between the computed commands $\hat{w}$ and the applied commands $w$ are depicted in Fig. 17. Also in this case, because the upper-bounds (13) on the time-delay are always fulfilled, the applied commands $w$ are just a translated version of $\hat{w}$, that is $w(t) = \hat{w}(t - \tau_f, t)$.

5. Conclusions

In this paper the problem of supervising multi-area power systems subject to LFC requirements and bounded time-delays on the communication channels has been considered. Detailed investigations have been undertaken on the effect of communication time-delay of the coordination performance. Centralized master/slaves strategies have been presented and discussed and their relevant properties summarized. Special efforts have
been devoted at investigating how the proposed strategy behaves under critical events as faults, failure or abrupt changes in the load demand.

The simulations have shown that the proposed strategy is capable to handle hard-to-predict adverse events whereas standard ALFC schemes result to be ineffective. In particular, the proposed approaches ensure viable evolutions to the overall networked system with respect to safety and operative constraints, despite remarkable changes in the load demand, in the presence of faulty situations and communication time-delays.

References


